Notation

Hiding constants

Unless explicitly stated otherwise, $O(\cdot)$-notation hides absolute multiplicative constants. Concretely, every occurrence of $O(x)$ is a placeholder for some function $f(x)$ that satisfies $\forall x \in \mathbb{R}. \, |f(x)| \leq C|x|$ for some absolute constant $C > 0$. Similarly, $\Omega(x)$ is a placeholder for a function $g(x)$ that satisfies $\forall x \in \mathbb{R}. \, |g(x)| \geq |x|/C$ for some absolute constant $C > 0$.

Vectors

All vectors are column vectors unless specified otherwise. In particular, the notation (a, b, c) is short hand for a column vector with entries $a, b, c \in \mathbb{R}$,. We denote the coordinate basis of $\mathbb{R}^n$ by $\{e_i\}_{i \in [n]}$. For a vector $v \in \mathbb{R}^n$, we let $v^\intercal$ be the corresponding row vector.

Inner products and norms

For vectors $u, v \in \mathbb{R}^n$ with $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$, we define the inner product of u and v, unless specified otherwise,

$$\langle u, v \rangle = u^\intercal v = \sum_{i=1}^{n} u_i \cdot v_i \,. \tag{1}$$

The (Euclidean) norm of a vector v is $\|v\| = \langle v, v \rangle^{1/2}$. For $p \geq 1$, we define the ℓ^p-norm of v,

$$\|v\|_p = \left(\sum_{i=1}^{n} |v_i|^p \right)^{1/p} \,. \tag{2}$$

For $p = \infty$, we take the limit, so that

$$\|v\|_\infty = \max_{i \in [n]} |v_i| \,. \tag{3}$$

Kronecker product

For two matrices A and B, their Kronecker product is the matrix A$\otimes B$ with entries $(A \otimes B)_{ii',jj'} = A_{i,j} B_{i',j'}$. This operation also applies to row and column vectors (viewed as matrices with only one column or one row). We use the notation $A^{\otimes k} = A \otimes \cdots \otimes A$ (k-times) for the k-fold tensor power of a matrix A.

Matrices

For matrices with more than two indices, we separate row and column indices by a comma. For example if A is a linear combination of matrices of the form $e_i(e_j \otimes e_k)^\intercal$, we denote the entries of A by $A_{i,jk}$. (Note that this convention is consistent with the above notation for Kronecker products.)

Traces

The trace is cyclic, that is, for all matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times m}$,

$$\operatorname{Tr} AB = \operatorname{Tr} BA\,. \tag{4}$$

A consequence of this property is that for $x, y \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$,

$$\operatorname{Tr} Axy^\intercal = \operatorname{Tr} y^\intercal Ax = \langle y, Ax\rangle\,. \tag{5}$$

Polynomials

Let $\mathbb{R}[x]$ be the set of polynomials with real coefficients in variables $x = (x_1, \ldots, x_n)$. For $d \in \mathbb{N}$, let $\mathbb{R}[x]_{\leq d}$ be the set of polynomials of degree at most d.

References